2023 IEEE Symposium Series on Computational Intelligence (SSCI)
Mexico City, Mexico. December 5-8, 2023

Using Graph Theory to Produce Emergent Behaviour in Agent-Based Systems

Brandon Gower-Winter
Department of Computer Science
University of Cape Town
Cape Town, South Africa
GWRBRA001@myuct.ac.za

Geoff Nitschke
Department of Computer Science
University of Cape Town
Cape Town, South Africa
gnitschke@cs.uct.ac.za

***Abstract*—Cooperation is a defining trait of Multi-Agent Systems. At the centre of these systems lies a communication network which governs how information flows from one agent to the next. However, the design of these networks is often overlooked despite the profound impact it can have on both the task performance of the agents and the emergent phenomena they produce. In this work we aim to illustrate this by investigating whether network centrality impacts the task performance and emergent inequality (unequal distribution of resources) of resource gathering agents. We achieve this by constructing several communication networks with increasing centrality and use them with an Agent-Based Model called *GATHER*. Our results indicate that as the variance of the population's centrality increases, the task performance of an agent population will decrease. Furthermore, we demonstrate that simply changing the centrality of the network can produce distinct results and emergent phenomena (inequality or the lack thereof in our case). We then further support this claim by increasing the reciprocity of one of our communication networks which results in a system with greater task performance and significantly lower inequality, further illustrating the impact communication network topology can have on Multi-Agent Systems.**

***Index Terms*—Agent-Based Model, Graph Theory, Centrality, Emergent Inequality, Reinforcement Learning**

I. Introduction

Cooperation, emergent or deliberate, is a defining characteristic of Multi-Agent Systems (MAS) [1] and more generally, collective behavior systems [2]. Whether in Robotics [3], Social Simulation [4]–[6] or MAS more generally [7], [8], Communication, Consensus and Social Networks are commonly at the heart of these systems and play a fundamental role in dictating how agents receive, perceive and communicate information to or from other agents.

More specifically, agent-to-agent interactions and communication are regulated by a network topology, the design of which may have a significant, often unpredictable, impact on the results produced by MAS. For example Wang et al. [9] demonstrated that even small perturbations in network topology can affect the number of cooperators in Prisoner's Dilemma games. Furthermore, Fontanari and Rodrigues [10] showed that when optimizing NK fitness landscapes, high connectivity and centrality boosts performance in smooth landscapes while slowing down information transmission (lower connectivity and centrality) boosts performance in large populations optimizing rugged landscapes. Given this, it is perhaps surprising to find that outside of consensus optimization [11]–[13], network topology is often overlooked in MAS design, thus providing an avenue for potentially fruitful research. One such endeavour is the study of network topology using Graph Theory metrics such as centrality and transitivity and how they relate to task (group) performance. Relevant works in this field include Vital and Martins [14] who investigated whether information flow of social animal groups could be inferred from graph metrics and Reia et al. [15] who found that groups that maximize the variance of their betweeness maximize their group performance for complex tasks. However, current work is limited by the fact that the effects (if any) these properties have on communication networks is unknown.

In this work we investigate centrality (degree of connectedness in a graph) and its effects on the task performance of a group of resource gathering agents. We are also interested in investigating emergent inequality (uneven distribution of collected resources among agents) in these systems. The motivation being that it is known that highly centralized nodes are the most affected by perturbations (spillover effects) [16]. This manifests as increased inequality in financial networks [17], but it is unclear if this trend persists in communication or information exchange networks more generally.

In short, this paper seeks to utilize an Agent-Based Model (ABM) to investigate *whether network centrality impacts the task performance and inequality of resource gathering agents*. We hypothesize that as centrality increases, task performance will decline and inequality will rise. The motivation being that in a highly centralized network, few agents will receive a majority of the information exchanged, thus allowing them to utilize this knowledge surplus to gather more resources than the other agents. The primary contribution of this work is the further illumination of the role the structure of a communication network has on MAS [15]. More specifically we highlight the role centrality plays on task performance as well as emergent inequality. The rest of the paper is presented as follows: Section II outlines the background work, Section III introduces the resource collection task, Section IV details our experiments and results and Section V concludes the paper and presents several avenues for future work.

II. Background

In MAS, communication networks come in many forms. For example, the vertices and edges of a network may be static (unchanging) [8], [15] or dynamic at some temporal scale [18], [19]. Information communicated along edges may be directional [20] and weighted [19], [21]. Furthermore, the presence of the network itself may be explicit [8], [15], [18], [19] or it may be implicit in the design of the system as is often the case in cooperative behaviour studies [21], [22]. In this work, we utilize an weighted directed graph (Section II-A) to allow the agents to solve a resource gathering task formulated as a Reinforcement Learning problem (Section II-B).

A. The Communication Network

Consider a set of n agents $V = \{v_0, v_1, \ldots, v_n\}$. We define their social or communication network as a weighted directed graph $G = (V, E, M)$ where E is the set of edges which consists of ordered pairs of vertices (Equation 1):

$$E \subseteq \{(x, y) \mid (x, y) \in V^2\} \tag{1}$$

and M is the Adjacency Matrix that defines the strength of the edges (Equation 2):

$$M = \begin{pmatrix} 1.0 & M_{01} & \ldots & M_{0n} \\ M_{10} & 1.0 & \ldots & M_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{n0} & M_{n1} & \ldots & 1.0 \end{pmatrix} \tag{2}$$

where M_{ij} is the strength of the edge between the ith and jth agents and ordered pair $(i, j) \in E$. Note that because M describes the degree to which two agents may share information, we assume that agents are able to perceive their own state with certainty (i.e $M_{ij} = 1.0 \mid i = j$). Additionally, we assume that M_{ij} is a non-negative floating point number $\in [0.0, 1.0]$. Where 0.0 indicates no information sharing and 1.0 indicates full information sharing.

B. Resource Gathering Using a Utility Model

In this work, agents partake in a cooperative resource gathering task. More specifically, agents are tasked with searching a grid-world environment for resources. When they encounter a resource, they must then return said resource to their home base (See Section III for more details). This process continues until all of the environment's resources have been collected or a computational budget has been spent. The primary goal of the agents is then to gather as many resources as fast as possible.

The mechanism that facilitates agent decision making is formulated as a Reinforcement Learning (RL) problem. For this, we rely on a modified version of Panait and Luke's [21] Utility Model for Cooperative Foraging. We believe it worth mentioning that before choosing this formulization, we also investigated a more traditional probabilistic stochastic artificial ant algorithm (PSA) [22] against a random-walk (RW) baseline. The RL-based approach exhibited greater task performance (resources collected) than PSA and RW over a wide-range of input parameter values, thus motivating its selection.

In Panait and Luke's [21] Utility Model, a resource collection task may be viewed as two separate sub-tasks: (1) Starting at the home base and finding a resource (goal state) and (2) Starting at the location of a recently collected resource and returning it to the home base (goal state). With this delineation, each agent v is assigned a value $p \in \{home, food\}$ which describes which sub-task the agent is trying to solve. Furthermore, we define $s \in S$ to be the state of the agent (its location in the grid-world), a set of actions $A = \{UP, DOWN, LEFT, RIGHT\}$ which describe the agents' deterministic movement in the grid-world, $R(s)$ as the reward function and a Utility function $U_{v,p}(s)$ which describes the utility of some state s for an agent v. Note that U is distinct for each sub-task which, for the purposes of this work, allows the agents to optimize for both the $home$ and $food$ sub-tasks independently. To calculate the Utility of a given state, an agent will add its own opinions with those of other agents that it is connected to. This is done using Equation 3.

$$U_{v,p}(s) = \sum_{i \in V} M_{vi} \hat{U}_{i,p}(s) \tag{3}$$

where M is the adjacency matrix of the communication network (Equation 2) and $\hat{U}_{v,p}(s)$ is the Utility for some state s for agent v without taking into consideration the opinions of other agents. Lastly, we define a policy $\pi(S \to A)$ which maps states to actions and returns the action that maximizes the agent's utility $U_{v,p}(s)$ (Equation 4):

$$\pi(s) = \max_{a \in A} U_{v,p}(s') \tag{4}$$

where s' is the state of the agent after taking action a. After an agent has taken an action, it will update its personal Utility opinions $\hat{U}_{v,p}(s)$.

III. Methodology

To investigate the role centrality plays on task performance, we make use of an ABM called *GATHER*[1] developed using *ECAgent*[2] in Python 3. In *GATHER*, agents are placed in a grid-world environment and tasked with gathering resources. Each agent learns about their environment using the RL-based decision making framework outlined in Section II-B. The agents then propagate this information along a communication network so other (connected) agents may utilize each other's acquired knowledge.

[1]*GATHER* source code available at: https://shorturl.at/tvUVZ

[2]*ECAgent* framework available at: https://ecagent.readthedocs.io/en/latest/

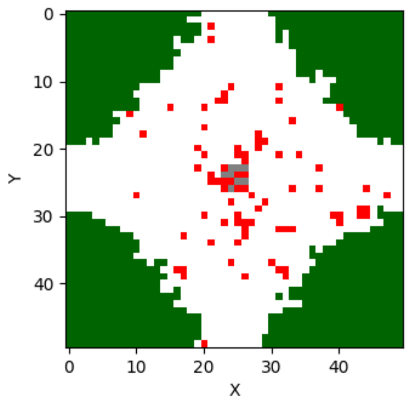


Fig. 1: A Visualization of the *GATHER* ABM at some abitrary timestep. Red pixels represent agents, green pixels are uncollected resources and grey pixels represent the home base.

At the beginning of a simulation run, all agents are placed at their home base, a 3×3 cell placed at the center of the environment. Each iteration, the agents move in one of the four cardinal directions in search of resources using Equation 4 to choose the direction to move in. When an agent finds a resource, it collects it and attempts to find its way back to the home base to deposit the resource. A resource is only considered gathered once it has been returned back to the home base. This process repeats for all agents until all resources have been collected or some computation budget (timestep limit) has been spent. Figure 1 shows a visual representation of what *GATHER* looks like. Note that *GATHER* does support randomly setting the location of the home base at the start of a simulation run as well as customizing the resource distribution of the environment. In this work, all simulation runs will have the home base placed at the center of the environment and will use the same resource distribution shown in Figure 1

As shown in Figure 1, once a resource is collected, it no longer exists in the environment. That is to say that the resource gathering task is dynamic and the applicability or usefulness of some of the agents' beliefs captured by the Utility function fluctuate over the course of a simulation run. Consider a scenario in which an agent associates a cluster of resources with a high Utility. Naturally, the agent will continue to move back and forth between this cluster and the home base as it tries to collect as many resources as possible, as fast as possible. Overtime, the Utility of the resource cluster diminishes as resources are collected by said agent (or other agents). Once a resource cluster is emptied, it should no longer possess any Utility , but this only occurs after the agent and other agents repeatedly return to this location and update their Utility ”maps” accordingly, a process that greatly slows down the resource collecting capabilities of the agents. To combat this, *GATHER* borrows from traditional pheromone-based or ant-inspired algorithms [21] and implements a type of Utility decay whereby each iteration, each agent's personal Utility opinions are decayed in accordance with Equation 5.

$$\hat{U}_{v,p,t+1}(s) = \lambda_p \hat{U}_{v,p,t}(s) \tag{5}$$

Where t is the timestep of the simulation and λ_p are the user-defined decay rate parameters with λ_{home} and λ_{food} the *home* and *food decay rate* parameters respectively.

We borrow another concept from ant-inspired MAS called ”ant mill prevention”. Ant milling is a phenomena whereby cooperative agents, performing different sub-tasks may get stuck in a feedback loop, following each other in perpetuity [22]. To prevent this, *GATHER* does two things: First, agents are not allowed to move to a grid-cell that they occupied on the previous iteration (i.e $s_{t-1} \neq s_{t+1}$). Second, if an agent occupies the same cell as two or more other agents, it elects to take a random action instead of the optimal action defined by its policy π. During development, we found that these two additional rules noticeably reduced the likelihood of unintended behaviour emerging during a simulation run.

In *GATHER*, an agent is rewarded on two occasions: (1) when it moves from an empty cell to a resource-rich cell during the resource search sub-task and (2) when it returns a resource to the home base when performing home search sub-task. These rewards form part of an agent's personal Utility update which are as follows (note that the v subscript is omitted for clarity): when a resource carrying agent is searching for the home base, Equations 6 and 7 are used:

$$\hat{U}_h(s) = \hat{U}_h(s) + \eta(r + \gamma(\hat{U}_h(s') - \hat{U}_h(s)) \tag{6}$$

$$\hat{U}_f(s') = \hat{U}_f(s') + \eta(r + \gamma(\hat{U}_h(s) - \hat{U}_f(s')) \tag{7}$$

where $\hat{U}_h(s)$ is an agent's personal Utility opinion of state s when performing the home search h sub-task. Conversely, $\hat{U}_f(s)$ is an agent's personal Utility opinion of state s when performing the resource search f sub-task. γ and η are user-defined input parameters for discount factor and learning rate respectively. Lastly, r is the reward and is equal to 1.0 if the agent completes the sub-task (i.e. finds the home-base while carrying a resource) and 0.0 otherwise.

If the agent is searching for resources, we use Equation 8:

$$\hat{U}_f(s) = \hat{U}_f(s) + \eta(r + \gamma(\hat{U}_f(s') - \hat{U}_f(s)) \tag{8}$$

where r is equal to 1.0 if the agent completes the sub-task (i.e. finds a resource cell while searching for a resource) and 0.0 otherwise. Lastly, we use a special rule (Equation 9) for when an agent is searching for food and does not find any:

$$\hat{U}_h(s') = \hat{U}_h(s') + \eta(1.0 + \gamma(\hat{U}_f(s) - \hat{U}_h(s')) \tag{9}$$

The motivation being that preliminary experiments showed that the agents where able to complete the resource gathering task more efficiently when they were constantly and collectively "signalling" the direction of the home base. It may also be noted that both Equations 7 and 9 propagate information from one Utility "map" to another. The motivation for this decision being that back and forward "updating" [21] has been shown to improve the learning efficiency of the agents as it not only allows the agents to learn both personal Utility "maps" simultaneously, it also allows useful Utility information to be shared between them.

IV. Experiments and Results

Recall that our overall goal is to investigate the role centrality plays at facilitating greater (or lesser) task performance in cooperative resource gathering agents. We achieve this by designing 6 communication networks that exhibit varying degrees of network centrality. When designing these networks, we had to consider which centrality metric to use. We opted for degree centrality because it has the highest correlation to other centrality metrics [23], thus potentially making our results more comparable to other related works using other centrality metrics.

For each communication network, a label $l \in [0.0, 1.0]$ is assigned. The value of this label denotes the proportion of agents that are fully-connected to all other agents in the environment, and therefore capable of perceiving their Utility opinions. The higher l, the greater the average degree centrality of the network. Agents that are not fully connected to the network are restricted to communicating with two other agents whose identification numbers lie just before or just after the agent in question (i.e. Agent with id $= 2$ is connected to agent 1 and agent 3. For clarity we measured the degree centrality of the networks using Equations 10 and 11.

$$C(v) = \frac{deg(v)}{N-1} \tag{10}$$

$$C_{pop}(V) = \frac{1}{N}\sum_{v \in V} C(v) \tag{11}$$

where $C(v)$ is degree centrality of an agent v, $C_{pop}(V)$ is the average centrality of an agent population V, $deg(v)$ is the degree of an agent and N is the size of the population of agents. The final l values used for each of the communication networks were $l \in \{0.0, 0.2, 0.4, 0.6, 0.8, 1.0\}$ and closely represent the average degree centrality of the communication network as a whole.

With the networks setup, we initialized *GATHER* environments of size 50×50 with $N = 100$ agents and ran each simulation for $t = 5000$ timesteps. Preliminary experiments investigated varying $N = \{10, 100, 1000, 10000\}$. We settled on the aforementioned value of $N = 100$ because fewer agents were unable to effectively solve the resource gathering task while larger quantities of agents could solve the resource gathering task without learning (i.e. by brute force due to sheer number of agents occupying the environment).

TABLE I: *GATHER* Initialization Parameters

Property	Value
Timesteps (t)	5000
Agents (N)	100
l	$\in [0.0, 0.2, 0.4, 0.6, 0.8, 1.0]$
λ_{home}	0.0
λ_{food}	0.01
η	0.005
γ	0.8

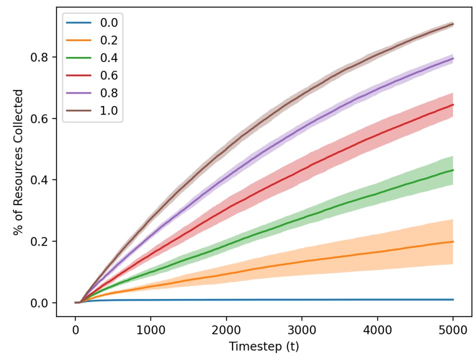


Fig. 2: Figure illustrating the average percentage of resources collected by the agents over an entire simulation run. The shaded regions represent an interval range of one standard deviation. The legend indicates which communication network (l) produced which result.

$t = 5000$ was chosen because the general performance of simulation runs with $N = 100$ could be reliably ascertained in that time frame.

A full list of chosen parameters in included in Table I and represents the results of a parameter tuning process. All chosen parameters (except λ_{home}) are resilient to slight perturbation. In the case of $\lambda_{home} = 0.0$, parameter tuning revealed that no Utility decay was the most beneficial. The results indicating that the stationary location of the home base means that any knowledge acquired of its location in the environment will be useful for the entire duration of a simulation run, which is not the case for the resource search sub-task as resource patches are depleted overtime. For each communication network in l, we ran 20 uniquely-seeded simulations and report the results below.

As shown in Figure 2, as the variance of centrality increases (decreasing l), so does task performance. Most notably, when there are no central agents ($l = 0.0$), the agents fail to collect many resources at all. This is expected as the population's communication capabilities are almost non-existent and the resource collection task requires some degree of cooperation (via communication) to be efficiently solved. Interestingly, populations with lower degrees of centrality ($l \leq 0.6$) exhibited higher variance in their task performance, suggesting that the presence of a significant

TABLE II: Results of a post-hoc Dunn test performed on the total resources collected (task performance) across all communication networks investigated (l).

(l)	0.0	0.2	0.4	0.6	0.8	1.0
0.0	$\sim$	$-$	$+$	$+$	$+$	$+$
0.2	$-$	$\sim$	$-$	$+$	$+$	$+$
0.4	$+$	$-$	$\sim$	$-$	$+$	$+$
0.6	$+$	$+$	$-$	$\sim$	$-$	$+$
0.8	$+$	$+$	$+$	$-$	$\sim$	$-$
1.0	$+$	$+$	$+$	$+$	$-$	$\sim$

A $+$ indicates a significant difference ($p = 0.05$) in results were found. Conversely, a $-$ indicates no significant difference was found.

number of highly connected agents results in stable resource collection behaviour in the overall agent population. Hutter et al. [24] have shown that centrality metrics can evolve stable behaviour. They attribute their finding to the resilience to misinterpretations offered by being highly central in a social network. Our results support this claim by demonstrating that high-degrees of centrality in a large portion of the agent population will produce more consistent emergent behaviour. This manifests as increased resilience to misinterpretation whereby highly-connected agents are able to make decisions based on a greater quantity of information provided by a greater number of connected agents. Conversely, agents with few connections must rely on, potentially incomplete, information provided by their connections.

To validate our claims, a Kruskal-Wallis H test ($p = 0.05$) was performed on the results produced by the different communication networks investigated. The results indicated a significant difference in the task performance between the different networks. We then conducted a post-hoc Dunn test ($p = 0.05$) to identify which pairs had significantly different results (See Table II). In general, our claim that increasing the quantity of highly-connected agents leads to greater task performance is supported. In fact, were the confidence interval set $p = 0.1$, the Dunn test would have produced significant results between all pairs.

We also examined the degree of emergent inequality produced by each network. These results are shown in Figure 3. Perhaps unsurprisingly, the inequality at the end of each simulation is inversely related to the task performance of the population of agents with $l = 0.0$ having the lowest task performance, highest inequality and $l = 1.0$ having the highest task performance and lowest inequality. Across all networks (except $l = 0.0$), the variance of this inequality is low. It is well known that disparities in information access produce inequality [16], [17] and our results support those findings. Interestingly, further investigation revealed that the primary reason for the increase (not presence) in emergent inequality was due to the disparity in the amount of resources collected by highly-connected agents compared to the agents with few connections. Our results indicate that it is common for highly connected agents to collect the vast majority of resources even when normalising for population differences.

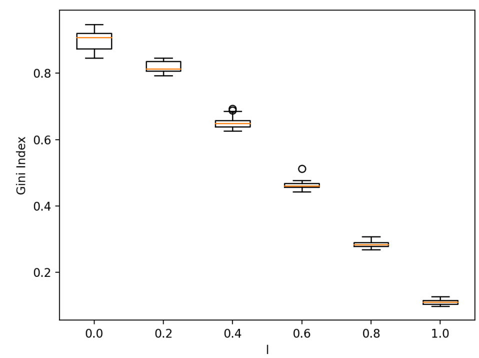
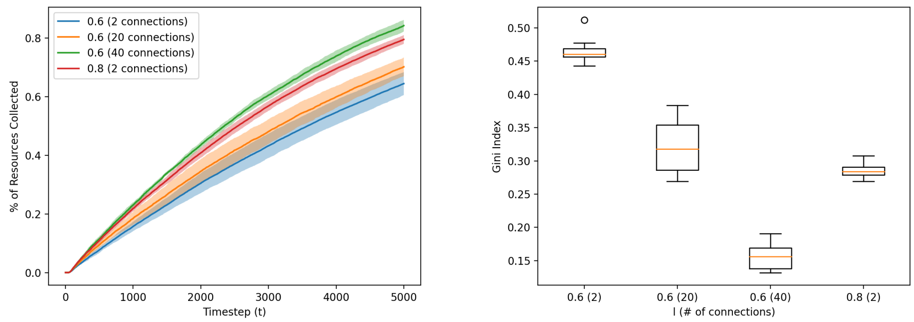


Fig. 3: Figure showcasing the final degree of inequality exhibited by each of the communication networks (l) investigated. A value closer to 1.0 indicates greater inequality.

Recall that each agent that is not fully-connected in our model only has two connections (the previous and next logical agents). What if that number were greater? We conducted two supplementary scenarios modifying the $l = 0.6$ network where instead of 2 connections, each partially-connected agent has 20 and 40 logical connections respectively. Note that these values were chosen for a particular reason, by increasing the number of logical connections the partially-connected agent have, we increase the reciprocity of the network, that is another graph metric which describes the likelihood of two vertices being connected to each other. With the values of 20 and 40, the average centrality of network increases but not past that of $l = 0.8$. The results of this process are shown in Figure 4a where we can see that task performance increases with a Dunn test ($p = 0.05$) revealing that the $l = 0.6$ (40 connections) network had significantly greater task performance despite having lower average centrality. Furthermore, Figure 4b illustrates that increased network reciprocity results in a significant reduction in emergent inequality as shown clearly by the $l = 0.6$ (20 connections) $l = 0.6$ and (40 connections) networks.

V. Conclusions and Future Work

Cooperation is a defining characteristic of MAS [1] and collective behavior systems [2]. At the heart of these systems are communication networks which play a fundamental role in dictating how agents receive, perceive and communicate information to or from other agents. In this work, we utilized an ABM to investigate whether network centrality impacts the task performance and inequality of resource gathering agents. We hypothesized that as centrality increases, task performance will decline and inequality will rise. For the most part, this hypothesis was correct. Our results indicated that as the variance of the population's centrality increased, the task performance decreased. Taking our primary results in tandem with our supplementary results, our work clearly demonstrates why the topology that governs a MAS cannot



(a) (b)

Fig. 4: Figures of task performance (a) and emergent inequality (b) for the supplementary experiments. The plots are labelled $x(y)$ where x is the l input parameter and y is the number of reciprocated connections for non-fully connected agents.

be overlooked. We showed that simply changing the centrality of the network can produce distinct results and emergent phenomena (inequality or the lack thereof in our case). We then further supported this claim by increasing the reciprocity of one of our communication networks. This resulted in a system with greater task performance and significantly lower inequality further illustrating that when taking multiple graph metrics into account, the types of behaviour a MAS may elicit increases too. Given the simple alterations made to the communication networks in this work, future endeavours will look at more complex graph metrics such as modularity and transitivity. We will also be evaluating these networks across multiple tasks to ascertain the applicability of each graph metric for specific tasks. The ultimate goal of this work being the pursuit of a system that organically evolves communication networks to aid MAS in solving tasks.

REFERENCES

[1] J. E. Doran, S. Franklin, N. R. Jennings, and T. J. Norman, "On cooperation in multi-agent systems," *The Knowledge Engineering Review*, vol. 12, no. 3, pp. 309–314, 1997.

[2] G. Nitschke, "Emergence of cooperation: State of the art," *Artificial Life*, vol. 11, no. 3, pp. 367–396, 2005.

[3] J. Hu, J. Xu, and L. Xie, "Cooperative search and exploration in robotic networks," *Unmanned Systems*, vol. 1, no. 01, pp. 121–142, 2013.

[4] S. J. Alam and A. Geller, "Networks in agent-based social simulation," *Agent-based models of geographical systems*, pp. 199–216, 2012.

[5] C. Perret, S. T. Powers, J. Pitt, and E. Hart, "Can justice be fair when it is blind? how social network structures can promote or prevent the evolution of despotism," in *ALIFE 2018: The 2018 Conference on Artificial Life*. MIT Press, 2018, pp. 288–295.

[6] K. Garg, C. Padilla-Iglesias, N. Restrepo Ochoa, and V. B. Knight, "Hunter–gatherer foraging networks promote information transmission," *Royal Society Open Science*, vol. 8, no. 12, p. 211324, 2021.

[7] S. D. Dionne, H. Sayama, and F. J. Yammarino, "Diversity and social network structure in collective decision making: evolutionary perspectives with agent-based simulations," *Complexity*, vol. 2019, 2019.

[8] D.-J. van Veen, R. S. Kudesia, and H. R. Heinimann, "An agent-based model of collective decision-making: how information sharing strategies scale with information overload," *IEEE Transactions on Computational Social Systems*, vol. 7, no. 3, pp. 751–767, 2020.

[9] S. Wang, M. S. Szalay, C. Zhang, and P. Csermely, "Learning and innovative elements of strategy adoption rules expand cooperative network topologies," *PloS one*, vol. 3, no. 4, p. e1917, 2008.

[10] J. F. Fontanari and F. A. Rodrigues, "Influence of network topology on cooperative problem-solving systems," *Theory in Biosciences*, vol. 135, pp. 101–110, 2016.

[11] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[12] M. Rafiee and A. M. Bayen, "Optimal network topology design in multi-agent systems for efficient average consensus," in *49th IEEE Conference on Decision and Control (CDC)*. IEEE, 2010, pp. 3877–3883.

[13] K. You and L. Xie, "Network topology and communication data rate for consensusability of discrete-time multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 56, no. 10, pp. 2262–2275, 2011.

[14] C. Vital and E. P. Martins, "Using graph theory metrics to infer information flow through animal social groups: a computer simulation analysis," *Ethology*, vol. 115, no. 4, pp. 347–355, 2009.

[15] S. M. Reia, S. Herrmann, and J. F. Fontanari, "Impact of centrality on cooperative processes," *Physical Review E*, vol. 95, no. 2, p. 022305, 2017.

[16] A. Antinyan, G. Horváth, and M. Jia, "Social status competition and the impact of income inequality in evolving social networks: An agent-based model," *Journal of Behavioral and Experimental Economics*, vol. 79, pp. 53–69, 2019.

[17] C. Ghiglino and S. Goyal, "Keeping up with the neighbors: social interaction in a market economy," *Journal of the European Economic Association*, vol. 8, no. 1, pp. 90–119, 2010.

[18] V. A. Folcik, G. Broderick, S. Mohan, B. Block, C. Ekbote, J. Doolittle, M. Khoury, L. Davis, and C. B. Marsh, "Using an agent-based model to analyze the dynamic communication network of the immune response," *Theoretical Biology and Medical Modelling*, vol. 8, no. 1, pp. 1–25, 2011.

[19] T. Sebestyén and A. Varga, "Knowledge networks in regional development: an agent-based model and its application," *Regional Studies*, vol. 53, no. 9, pp. 1333–1343, 2019.

[20] R. Vidgen and J. Padget, "Sendero: An extended, agent-based implementation of kauffman's nkcs model," *Journal of Artificial Societies and Social Simulation*, vol. 12, no. 4, p. 8, 2009.

[21] L. Panait and S. Luke, "A pheromone-based utility model for collaborative foraging," in *Proceedings of the Third International Joint Conference on Autonomous Agents and Multiagent Systems, 2004. AAMAS 2004.* IEEE, 2004, pp. 36–43.

[22] A. R. Cheraghi, J. Peters, and K. Graffi, "Prevention of ant mills in pheromone-based search algorithm for robot swarms," in *2020 3rd International Conference on Intelligent Robotic and Control Engineering (IRCE)*. IEEE, 2020, pp. 23–30.

[23] T. W. Valente, K. Coronges, C. Lakon, and E. Costenbader, "How correlated are network centrality measures?" *Connections (Toronto, Ont.)*, vol. 28, no. 1, p. 16, 2008.

[24] C. Hutter, R. Lorch, and K. Bohm, "Evolving cooperation through reciprocity using a centrality-based reputation system," in *2011 IEEE/WIC/ACM International Conferences on Web Intelligence and Intelligent Agent Technology*, vol. 2. IEEE, 2011, pp. 264–271.